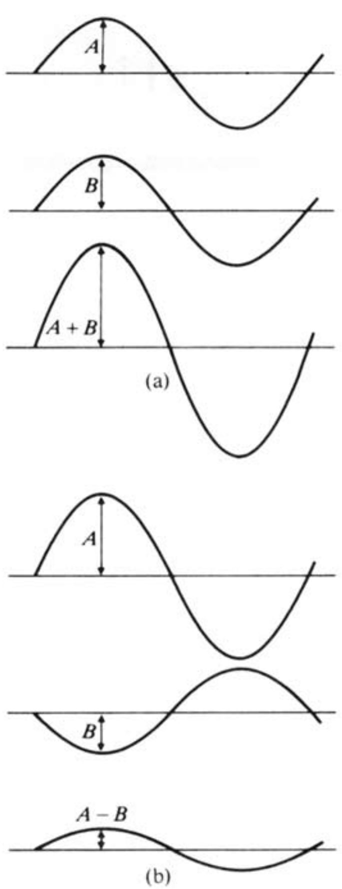

FIG. 8.1
Linear superposition of two simple harmonic motions at the same frequency: (a) same phase; (b) opposite phase.

vidual responses at each point in time. If two simple harmonic motions at the same frequency are superimposed, the resultant will also be a simple harmonic motion at that frequency. The amplitude of the resultant will depend not only on the amplitude of the components but also on the fractional part of the period through which each component has passed. This fraction of the period is known as the *phase*. Figure 8.1 illustrates two cases as examples.

In the first case, the two motions are identical in phase and the displacements add. In the second case, they are opposite in phase and the displacements subtract (if $A = B$, total cancellation is the result). The concept of phase is a useful one in the comparison of two simple harmonic motions as well as in their superposition. It merits discussion in some detail.

8.2 PHASE ANGLE

Simple harmonic motion can be described as the projection of uniform circular motion onto an axis, as shown in Fig. 8.2. As the circle rotates, the projection of point P on the y-axis moves in simple harmonic motion. The angle ϕ indicates how far the circle has turned, so as ϕ increases, the corresponding point P moves to the right on the graph of displacement vs. time. During one complete revolution ϕ increases by 360°, and the point moves to the right a distance T on the time axis (T is the period of the motion in seconds).

The representation of simple harmonic motion as the projection of circular motion can be demonstrated in several ways, one of which is shown in Fig. 8.3. The shadow of a wheel with a crank is projected alongside a mass vibrating in simple harmonic motion at the end of a spring (see Section 2.1). If the speed of the wheel is adjusted so that the time required for one revolution is the same as the period of the mass-spring vibrator, its shadow will move up and down in synchronism with the mass. Both the mass and the shadow of the crank move up and down in simple harmonic motion.

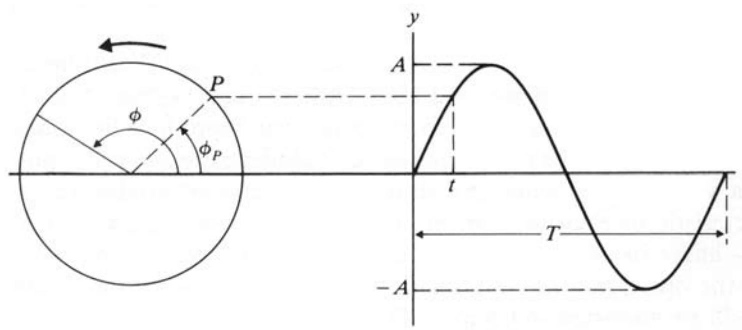

FIG. 8.2
Simple harmonic motion represented as the projection on a vertical axis of the point P moving around a circle at a uniform rate.

Simple harmonic motion is often referred to as sinusoidal motion, because the projection of circular motion on the y-axis is given analytically by a trigonometric function called the sine; that is,

$$y = A \sin \phi = A \sin 360 \frac{t}{T},$$

where the amplitude of the vibration A also equals the radius of the circle projected. Also, 360 is the number of degrees in a complete circle, t/T is the fraction of a complete circle subtended by angle ϕ in time t, T is a complete period, and sin is the abbreviation for sine. The language of trigonometry will not be used in this book, however.

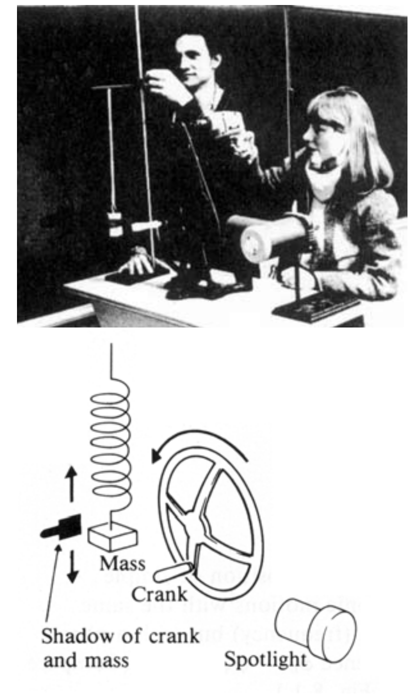

FIG. 8.3
Demonstration of simple harmonic motion as the projection of circular motion. By rotating the wheel at the proper rate, the shadow of a crank can be made to move up and down in synchronism with a mass-spring vibrator. (Photograph by Christopher Chiaverina).

We now add a second point in simple harmonic motion, such as point Q in Fig. 8.4. The projection of point Q moves with the same period T and amplitude A as point P, but it obviously has a different motion. At any given time t, points P and Q will be at different points on the circle. Thus the projected points reach their maximum and minimum values at different times. At any given time t, the positions of ϕ_P and ϕ_Q on the two curves result from different angular positions and ϕ_Q of the rotating points. The difference $\phi_P - \phi_Q$, which remains constant, is called the phase difference between the two simple harmonic motions.

The complete description of a given simple harmonic motion requires three parameters: the period (or frequency), the amplitude, and the initial phase. In the case of sound, phase has significance only when it is used to compare two or more waves or vibrations.

8.3 COMBINATION OF TWO SIMPLE HARMONIC MOTIONS

In order to radiate a pure tone, a loudspeaker cone moves in and out with a motion that is essentially simple harmonic motion. How does

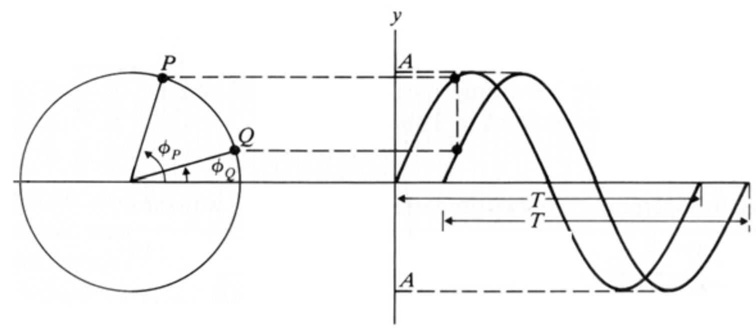

FIG. 8.4
Two points P and Q move with the same period T and amplitude A, and maintain a constant phase difference $\phi_P - \phi_Q$.